

# Expanding Universe and its manifestations beyond the General Relativity.

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## Abstract

It has been demonstrated that a modern stage of the Universe expansion may be described in accordance with the observations within the scope of the space-time conformal geometry. The clock synchronization procedure in SR has been generalized to the case of the expanding space. It has been found that a universal local manifestation of the cosmological expansion is a background acceleration, the value of which is determined by Hubble constant. The formulae defining an explicit red-shift dependence of the cosmological distance and expressions for Hubble law have been obtained in a pure kinematic way from the conformal group transformation, providing a quantitative representation of the Pioneer anomaly and of the effect associated with the experimentally revealed Metagalaxy transition to its accelerated expansion.

## 1 Introduction

The existence of the experimentally recorded local manifestations of a cosmological expansion unexplained by a conventional gravitation theory has received much thought since the discovery of the Pioneer anomaly (PA). It is well known that this phenomenon - systematic violet frequency shift detected by radio signals of the Pioneer10/11 space craft - still calls for a consistent explanation [1-4]. It is customary to interpret the observed

effect as a result of an additional uniform acceleration ( $a_p$ ) of the space craft, calculated as  $a_p = (8.74 \pm 1.33)10^{-8}cm/c^2$  and directed (approximately) toward the Sun. A measurand in this case is the propagation-time proportional frequency shift  $\Delta\nu_a$  in a signal from the transmitter located at each space craft. The experimentally recorded uniform rate of this shift  $\Delta\dot{\nu}_a = \frac{d(\Delta\nu_a)}{dt}$  is equal to [1,2]

$$\Delta\dot{\nu}_a = (5.99 \pm 0.01)10^{-9}Hz/s. \quad (1.1)$$

The shift direction observed is in line with a decrease in the expected red shift (as a transmitter recedes from the observation point), i.e. the shift is actually violet and anomalous by its nature.

According to the conventional treatment based on a natural supposition concerning the Doppler origin of the shift, this effect may be explained, with regard to a nonrelativistic character of a relative motion of the object, as follows.

A red frequency shift of the signal  $\nu_{obs}$  from the transmitter operating at the frequency  $\nu_0$  and receding from the observation point with the speed  $V \ll c$  is determined by the well-known formula from a theory of Doppler effect <sup>1</sup> as

$$\nu_{obs} = \nu_0 \left(1 - V/c\right). \quad (1.2)$$

With a negative uniform acceleration that is equal to  $-W_a$ , the expression for speed is of the form

$$V = V_0 - W_a t, \quad (1.3)$$

where  $V_0$  – constant speed component. From this we have

$$\Delta\nu_a \stackrel{def}{=} \nu_{obs} - \nu_0 \left(1 - V/c\right) = \nu_0 \frac{W_a}{c} t. \quad (1.4)$$

Consequently, the rate  $\dot{\nu}_a$  of the anomalous violet radiation-frequency drift equals

$$\dot{\nu}_a = \nu_0 \frac{W_a}{c}. \quad (1.5)$$

Comparing (1.5) with (1.1), a numerical value of the anomalous acceleration  $W_a$  may be determined from

$$W \cong c/\nu_0 6 \cdot 10^{-9} cm/s^2,$$

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<sup>1</sup> Here and hereinafter the consideration is given to the radial speed component and hence to the longitudinal Doppler effect.

where  $\nu_0 = 2.29 \cdot 10^9 Hz$  - operating frequency of the Pioneer 10/11space craft transmitter. Hence it follows that  $W \cong 7.86 \cdot 10^{-8} cm/s^2$ , coincident with the above-mentioned acceleration  $a_p$  within the limits of a permissible value.

Obviously, the treatment considered necessitates the existence of the dynamic factors really causing such acceleration. But it has been impossible to explain adequately this effect due to the detection of the corresponding gravitating sources (see [1-4]). As noted in [1], all the efforts were in vain *facing a stiff experimental wall*.

Quite natural in such a situation is to search for alternative approaches to possible physical causes of the observed effect. One of the approaches is associated with the assumption that there exists some time “inhomogeneity” analytically represented as a nonlinear function  $t'(t)$ , where  $t$  denotes “homogeneous” time. Then we can introduce the notion of “clock acceleration” (see [1,2])  $\alpha_t$  as follows:

$$\alpha_t \stackrel{def}{=} \frac{d^2 t'}{dt^2} \text{ (dimension} - (\text{time})^{-1}\text{)}. \quad (1.6)$$

For  $\alpha_t$  to be constant, nonlinearity must be quadratic that, in combination with the apparent requirement  $\left(\frac{dt'}{dt}\right)_{t \rightarrow 0} \rightarrow 1$ , leads to the following function:

$$t'(t) = t + \frac{\alpha_t}{2} t^2. \quad (1.7)$$

As the expected nonlinearity effect should be small, we assume fulfillment of the condition  $\alpha_t t \ll 1$ .

It is easily seen that such an *ad hoc* hypothesis in fact explains the observed frequency shift without having recourse to the existence of an additional acceleration for the signal source. Indeed, let  $\Delta t$  and  $\Delta t'$  be respectively the times of the emitted and received signals, considered as a finite wave train containing a particular fixed number ( $N$ ) of complete oscillations. In the process it is assumed that both time intervals  $\Delta t$  and  $\Delta t'$  are considerably less than the signal propagation period ( $t$ ). The frequencies of the emitted ( $\nu_0$ ) and received ( $\nu_{obs}$ ) signals are determined from

$$\nu_0 = \frac{N}{\Delta t}, \nu_{obs} = \frac{N}{\Delta t'}.$$

Based on (1.7) and with due regard for the requirements  $\Delta t, \Delta t' \ll t$ , we get

$$\Delta t' = \Delta t (1 + \alpha_t t),$$

from where we have

$$\nu_{obs} = \nu_0 (1 + \alpha_t t)^{-1} = \nu_0 (1 - \alpha_t t).$$

From this equation for the frequency shift  $\Delta \nu_0 \stackrel{def}{=} \nu_{obs} - \nu_0$  we derive the formula

$$\Delta \nu_a = -\nu_0 \alpha_t \cdot t,$$

with which an expression for the frequency shift rate  $\dot{\nu}_a$  may be determined by

$$\dot{\nu}_a = \frac{d(\Delta \nu_a)}{dt} = -\nu_0 \alpha_t. \quad (1.8)$$

Comparing (1.8) with the experimental value of (1.1), we can determine a numerical value for  $\alpha_t$  from

$$\alpha_t = \nu_0^{-1} \cdot 6 \cdot 10^{-9} s^{-1} = \frac{6}{2.29} \cdot 10^{-18} s^{-1} \cong 2.62 \cdot 10^{-18} s^{-1}.$$

A numerical value of the anomalous acceleration  $W_a$  is given by  $W_a = c\alpha_t$ , where  $c$  is the speed of light. It is clear that in this approach the quantity  $W_a$  could not be treated as a *real* (or caused by dynamic factors) acceleration. It is rather a matter of its specific imitation: acceleration mimicry of a kind.

A very intriguing fact of the closeness between the numerical value of  $\alpha_t$  and Hubble constant determined from the astrophysical data has been noted by several authors, beginning from the research group that had discovered (see [1,2]). And the efforts to relate the quadratic time nonlinearity to cosmological expansion followed almost immediately.

The fact that the actual electromagnetic-signal propagation time ( $t'$ ) in the expanding space should be “shifted” relative to the conventionally assumed time ( $t = \frac{1}{2} \{t^{fin} - t^{in}\}$ ) according to

$$t' = t - \frac{H_0}{2} t^2, \quad (1.9)$$

where  $H_0$  – Hubble constant, has been first demonstrated in [5] and supported subsequently in [6].

Actually, in [5,6] it has been found that in any Friedmann-type model (Robertson-Walker metric  $dS^2 = c^2 dt^2 - \chi^2(t) dr^2$ ) on condition  $\frac{d\chi}{dt} > 0$  (expanding space) an expression for the light signal propagation time in the approximation, linear with respect to Hubble constant, is automatically containing the quadratic nonlinearity.<sup>2</sup> The approach associated with a treatment of PA as a local manifestation of the cosmological expansion has been developed in a number of works (e.g., see [8-10] and the references), where the situation is treated in terms of the “clock acceleration”.

As it has been first shown in [11], the quadratic nonlinearity of the form (1.7) with  $\alpha_t = -H_0$  arises naturally within the scope of the clock synchronization procedure generalization by Einstein-Poincaré to the case of the expanding space. Then (see [12,13]) it has been found that this nonlinearity originates in the assumption  $H_0 t \ll 1$  as a consequence of the conformal time inhomogeneity. By the approach based on the notion of time inhomogeneity, an anomalous violet electromagnetic-radiation frequency drift in the location-type experiments is interpreted as a universal local kinematic effect caused by the cosmological expansion. This effect is independent of the presence of some real gravitating sources, and in the appropriate experimental conditions, in principle, it should be observed at any frequencies. From this viewpoint, PA is the first actually recorded effect of this kind. Practical detection of the presence (or of the absence) of an anomalous violet shift in other (e.g., optical) frequency range may contribute much to elucidation of the PA effect physical nature. An idea of the corresponding experiment has been put forward in [14].

This paper presents elementary theoretical considerations to substantiate the fact that the anomalous frequency shift ( $\Delta\nu_a$ ) is determined by

$$\Delta\nu_a = \nu_0 H_0 t, \quad (1.10)$$

where  $\nu_0$  – radiation frequency of a steady source,  $H_0$  – Hubble constant,  $t$  – electromagnetic signal propagation time, whereas the, experimentally detected near the red shift value  $z_{\text{exp}} = 0.46 \pm 0.13$ , transition from a decelerated Universe expansion to the accelerated one may be derived as a kinematic inference from the conformal space-time geometry.

Among the problems considered, we can name the following:

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<sup>2</sup> Note, however, that the treatment within the scope of GR based on the Schwarzschild solution at the background of Friedmann’s metric results in the quadratic function of Hubble constant (see [7]), and this is considerably lower than the observed value.

- Relationship between the process of the Pioneer 10/11 space craft tracking and the clock synchronization procedure in SR.
- Generalization of the clock synchronization procedure by Einstein-Poincaré to the case of the expanding space and quadratic time inhomogeneity.
- The background acceleration effect and observer's noninertial reference system (RS).
- Derivation of an expression for Hubble law as a kinematic inference from the conformal space-time geometry. Consideration of the universal violet frequency shift in the location-type experiments.
- Nonlinear deviations from Hubble law and the accelerated expansion effect without the dark energy.

## 2 Tracking of the Pioneer 10/11 space craft and clock synchronization procedure in SR

Each communication session with a space craft enabling measurements of its flying path parameters by observations from the Earth is organized as follows.

At the initial time  $t^{in}$ , set by the tracking station clock, a radio signal is transmitted in the direction of a receding space craft, and, being within the reach of this space craft, the signal initiates an airborne transmitter operating at the frequency  $\nu_0 = 2.29109$  Hz. The transmitted signal is received at the Earth in time  $t^{fin}$  by the station clock. The time interval

$$t = \frac{1}{2} \{t^{fin} - t^{in}\} \quad (2.1)$$

is considered as a propagation time of the radio signal from the moment of its sending to the instant of time when the airborne transmitter is initiated. The distance  $R$  between the space craft and the observation point at time  $t$  indicated by the station clock may be calculated from

$$R = ct, \quad (2.2)$$

where  $c$  – signal propagation speed (i.e. speed of light). A speed of the receding space craft at the same time is determined from the magnitude of Doppler red shift for the operating frequency of a satellite transmitter.

Both these relations are in obvious agreement with the laws of electrodynamics and hence with the SR concept. In this manner we can find numerical values for two basic characteristics (position and speed) of the trajectory for each of the space crafts. On the other hand, it is well known that by SR the determination of synchronism between two spatially separated events in each inertial reference system (IRS) specified is based on the use of the idealized time synchronization procedure. This procedure includes the following basic elements:

- transmission of an instantaneous electromagnetic signal from the point A, at the instant of time  $t_A^\circ$  by the clock localized at this point, in a direction of the point B;
- arrival of the signal in the point B at time  $t_B$  by the clock located at this point, and its instantaneous retransmission in the direction of the point A;
- return of the signal to the point A in time  $t_A$  by the clock located at this point.

In this way we have the time interval

$$\Delta t_A = t_A - t_A^\circ \quad (2.3)$$

measured by the clock .

Next the clock B indications are set by two measured indications  $t_A^\circ$  and  $t_A$  of the clock so that *by definition* its going be synchronized with the clock in IRS specified. To this end, we *postulate* the coincidence requirement for the time intervals of the synchronizing signal propagation in the forward ( $t_{AB} = t_B - t_A^\circ$ ) and backward ( $t_{BA} = t_A - t_B$ ) directions. From this we get

$$t_B - t_A^\circ = t_A - t_B \quad (2.4)$$

representing, from the mathematical viewpoint, a linear equation for  $t_B$ . Its solution is of the form

$$t_B = \frac{1}{2} (t_A + t_A^\circ). \quad (2.5)$$

It is obvious that we obtain

$$t_{BA} = t_{AB} = \frac{1}{2} (t_A - t_A^\circ).$$

It is easily seen that the set of operations with the help of which a distance to the Pioneer 10/11 space craft has been determined in the process

of each communication session actually repeats the features of the idealized time synchronization procedure in SR. Indeed, after obvious identification of the symbols in (2.1) and (2.3) – (2.5)

$$t^{in} = t_A^{\circ}, \quad t^{fin} = t_A,$$

it is clear that the time interval  $t = \frac{1}{2}(t^{fin} - t^{in})$ , in terms of which we calculate a distance to the space craft, in fact may be determined as

$$t = t_B - t_A^{\circ} = t_A - t_B, \quad (2.6)$$

where  $t_A = t^{fin}$  – time of the signal arrival in the observation point,  $t_B$  – time of a signal emission by the space craft, determined by (2.5).

s regards the synchronizing signal speed  $c$  involved in formula (2.2) for the distance, according to Einstein (see [15]), it is initially defined as a ratio between the total distance covered by the signal in the forward ( $R_{AB}$ ) and backward ( $R_{BA}$ ) directions and the time interval  $t_A - t_A^{\circ}$  measured by the clock (average speed):

$$c = \frac{R_{AB} + R_{BA}}{t_A - t_A^{\circ}}. \quad (2.7)$$

Making an assumption, natural for the pair of the reciprocally stationary clocks, that  $R_{AB}$  and  $R_{BA}$  are coincident, we get

$$c = \frac{2R_{AB}}{t_A - t_A^{\circ}},$$

from where with regard to (2.5) we obtain

$$R_{AB} = |R_A - R_B| = c(t_A - t_B). \quad (2.8)$$

By raising (2.8) to the square, we arrive at the fundamental light similar interval of SR (light cone equation):

$$S_{AB}^2 = c^2(t_A - t_B)^2 - |R_A - R_B|^2 = 0. \quad (2.9)$$

Considering (2.6), it is clear that expression (2.2) used in calculations of a distance from the observation point to the space craft is practically coincident with formula (2.8) that, within the scope of SR, determines a distance covered by the signal in the *positive* generatrix direction of the light cone with a vertex at the reference point of the observer (point ). In

accordance with the requirement of (2.4), this distance is coincident with that covered by the signal in the *negative* direction of the generatrix for the light cone having its vertex at the point B.

The foregoing description of the time synchronization procedure repeats the first paragraph of Section I in the fundamental work of Einstein [15].

### 3 Clock synchronization in expanding space. Quadratic nonlinearity of time and background acceleration effect. Noninertial reference system.

Now we consider the situation when spatial scales are varying with time. Taking into account the structural correspondence between the real location-type experiment and Einstein's clock synchronization procedure, let us find out what changes are involved into this procedure by the spatial expansion. To this end, an expression for the expansion law is written in the following form:

$$R^{-1}(t) \frac{dR(t)}{dt} = H_0, \quad (3.1)$$

where  $R(t)$  – time-dependent Euclidean distance,  $H_0$  – dimensional numerical constant  $(time)^{-1}$ .

After an elementary integration of equation (3.1), for the function  $R(t)$  we have

$$R(t) = R(t_0) \exp \{H_0 (t - t_0)\}, \quad (3.2)$$

where  $t_0$  – arbitrary but fixed instant of time. From this formula the expressions for distances covered by a signal in the forward ( $R_{AB}$ ) and backward ( $R_{BA}$ ) directions may be written as

$$\left. \begin{aligned} R_{AB} &= R(t_A^\circ) \exp \{H_0 (t_B - t_A^\circ)\} = R(t_B), \\ R_{BA} &= R(t_B) \exp \{H_0 (t_A - t_B)\} = R_{AB} \exp \{H_0 (t_A - t_B)\} \end{aligned} \right\}. \quad (3.3)$$

Then it is seen that

$$\frac{R_{BA}}{R_{AB}} = \exp \{H_0 (t_A - t_B)\} > 1, \quad (3.4)$$

i.e. the distances  $R_{AB}$  and  $R_{BA}$  may be coincident in the absence of expansion only (or for  $H_0 = 0$ ). On the assumption that the speed of light is independent of the source's rate of motion and of the signal propagation direction, the location distances covered by the signal in the forward and backward directions may be determined from

$$R_{AB} = c(t_B - t_A^\circ), \quad R_{BA} = c(t_A - t_B). \quad (3.5)$$

By substitution of (3.5) into (3.4) we arrive at

$$t_A - t_B = (t_B - t_A^\circ) \exp \{H_0(t_A - t_B)\}. \quad (3.6)$$

As seen, the signal propagation time in the backward direction is always in excess of that in the forward direction. Besides, expression (3.6) representing the rate synchronism requirement for clocks and is now a transcendental equation for  $t_B$ , as opposed to the previous case when the corresponding equation (2.4) was linear.

Moreover, the equation includes the constant  $H_0$  with the backward time dimension that determines the expansion rate. It is clear that the presence of such a constant sets a certain time scale  $t_{lim} \sim H_0^{-1}$ . Because of this, the location procedure of clock synchronization leads to the results rather close to the stationary case in SR but on the assumption of the constant  $H_0$  smallness, i.e. when the signal propagation time  $\Delta t$  in both directions meets the condition  $\Delta t H_0 \ll 1$ . Also, it should be noted that in the fundamental work of Einstein [15] the derivation of Lorentz transformations from the synchronization requirement (2.5) has been based on considerations concerning differentially small distances between the clocks.

So in expression (3.6) we limit ourselves to the terms linear with respect to  $H_0(t_A - t_B)$  in the exponential expansion. Then we have

$$t_A - t_B = (t_B - t_A^\circ) \{1 + H_0(t_A - t_B)\} \quad (3.7)$$

representing the following quadratic equation for  $t_B$ :

$$t_B^2 - \frac{2}{H_0} \left\{ 1 + \frac{H_0}{2}(t_A + t_A^\circ) \right\} t_B + \frac{1}{H_0} (t_A + t_A^\circ) + t_A t_A^\circ = 0. \quad (3.8)$$

Its roots are of the form

$$t_B^{(\pm)} = \frac{1}{H_0} + \frac{t_A + t_A^\circ}{2} \pm \frac{1}{H_0} \left\{ 1 + \frac{H_0^2}{4} (t_A - t_A^\circ)^2 \right\}^{\frac{1}{2}}.$$

Obviously, the solution, linear for  $H_0$ , that could be transformed to  $t_B = \frac{1}{2}(t_A^\circ + t_A)$  at  $H_0 \rightarrow 0$  is associated with a lower sign.

In this way the expression for  $t_B$  determined from the clock synchronization location as an explicit function of the initial and final indications of the “reference” clock is of the form

$$t_B = \frac{1}{2}(t_A^\circ + t_A) - \frac{H_0}{8}(t_A - t_A^\circ)^2. \quad (3.9)$$

Next, we determine the signal propagation time in the forward and backward directions as

$$t_{AB} = t - \frac{H_0}{2}t^2, \quad (3.10)$$

$$t_{BA} = t + \frac{H_0}{2}t^2, \quad (3.11)$$

where  $t = \frac{1}{2}(t_A - t_A^\circ)$  – propagation time in the absence of expansion.

As follows from (3.10), in fact, a time interval needed for a signal to reach the synchronized clock is less than  $t = \frac{1}{2}(t_A - t_A^\circ)$  by  $\delta t = \frac{H_0}{2}t^2$ , whereas expression (3.10) *per se* is coincident with relation (1.9), the use of which, as we have seen in point 1, leads to the practically observable anomalous violet shift.

Thus, the expansion violates the fundamental synchronism requirement for the spatially distant clocks (i.e. synchronism of the spatially separated events) assumed in SR. An analytical relation determining the indication of the clocks synchronized by two indications of the “reference” clock, on retention of the light-speed constancy postulate, in the general case turns out to be nonlinear, containing in the explicit form a constant of dimension – Hubble constant. In the approximation  $H_0 t \ll 1$  the corresponding nonlinearity is quadratic, directly leading to formula (1.10) for the anomalous frequency shift. By this means the quadratic inhomogeneity of time specially postulated as a possible source of PA (see [1,2]) proves to be a natural corollary from generalization of the basic clock synchronization procedure in SR for the expanding space. Note that in this case a quantitative (sign including) simulation of the directly observable effect (frequency shift) is attained using no indication of the existent acceleration of the signal transmitter.

If on the basis of (3.10) and (3.11) we return to definition (3.5) for the distances covered by the signal in the forward ( $R_{AB}$ ) and backward ( $R_{BA}$ )

directions, the following expressions may be obtained

$$R_{AB} = R_0 - \frac{Wt^2}{2}, \quad R_{BA} = R_0 + \frac{Wt^2}{2}, \quad (3.12)$$

where  $R_0 = ct$ ,  $t = \frac{1}{2}(t_A - t_A^\circ)$ .

$$W = cH_0 \quad (3.13)$$

Proceeding from the Galilean-Newtonian kinematics, the situation looks as though in the process of synchronization the clock B were displaced relative to the clock A at a uniform acceleration equal  $cH_0$  in the direction of point . Because of this, we can obtain an expression for the anomalous frequency shift of the form given in (1.4), from which we can get formula (1.10) by substitution of  $W = cH_0$  from (3.13). It is important that the “acceleration” thus obtained is not an acceleration in a dynamic sense, i.e. it has no relation to the existence of some real sources of force effects on the associated test bodies.

At the same time, involvement of the parameter  $W = cH_0$  having the dimension of acceleration in formulae (3.12) and (3.13) for distances points to the fact that the “extended” reference system, where the clocks are synchronized, is noninertial. Whereas a uniform “acceleration” (3.13), in principle, must be recorded at every spatial point of the observer’s reference system, i.e. it must be background in character. From this viewpoint, the anomalous violet frequency shift may be considered as experimentally recorded manifestations of the reference system noninertiality for a fixed observer. This fact has been noted, specifically in [6], where the corresponding mechanical analog (Foucault pendulum) has been indicated to demonstrate noninertiality of the Earth as a reference system. By author’s opinion, in the case under study a still better (optical) analogy is represented by the well-known Sagnac effect: experimental recording of a proper rotation of the Earth with the use of Michelson interferometer.

Of course, origination of the background acceleration  $W = cH_0$  may be interpreted, proceeding from the equivalence principle, as the presence of a constant background gravitational field. And the observable frequency shift  $\Delta\nu/a$  may be described in terms of the Einstein’s gravitational frequency shift. It is easily seen that in the considered case this shift is precisely violet as, in accordance with a negative acceleration sign, the effective gravitational potential at the observation point is always higher than that at the point of the signal emission.

So, we can see that, in conditions of the expanding space, the use of the clock synchronization location procedure by Einstein-Poincaré that forms the basis for SR actually results in the quadratic inhomogeneity of time, with a scale determined by the expansion rate (numerical value of Hubble constant).

It is acknowledged that in a history of physics and in history on the whole it is useless to speculate on how it could have been if. . . Nevertheless, Einstein could have received the above-mentioned result and hence could approach the prediction of the presence of a universal anomalous violet frequency shift should the Universe expansion be discovered not in 1929 but as early as the beginning of the first decade of the XX-th century when SR, as opposed to GR, has been already framed, and what is more in two versions.

Besides, conformal invariance of Maxwell electrodynamics established by Batmann [16] and Kanningham [17] in 1909-1910 has opened up possibilities for derivation of an analytical expression for Hubble law in a pure kinematic way on the basis of space-time transformations of the conformal group  $SO(4,2)$ . In the process the quadratic nonlinearity of time arises in the first approximation as a consequence of its initial conformal inhomogeneity.

## **4 Hubble law and violet frequency drift as a kinematic consequence of conformal space-time geometry.**

As is known, an experimental checking of the law of cosmological expansion calls for an independent determination of the galactic objects recession speed and of the distances from each of these objects to the observation point. Electromagnetic signals arriving in the observation point from cosmologically distant sources provide most important information.

The whole totality of modern astrophysical data bears witness to a vanishingly small spatial curvature of the observable Universe and to an approximate constancy of Hubble parameter, at least in the interval of values up to the red shift on the order of unity. In these conditions a non-Euclidean character of the space-time manifold is actually exhibited in inhomogeneity of time only. Because of this, it seems natural to attempt description of the electromagnetic signal propagation in conditions of a

modern Metagalaxy, retaining an Euclidean character of 3-D space and using some minimal extension of the standard relativistic kinematics of SR due to the introduction of a group of special conformal transformations changing the space-time scales.

Within the scope of this approach, an expression for Hubble law in the assumption of the Hubble parameter constancy  $H_0 = H(t_{today}) = \text{const}$  may be written as

$$V/R_L = H_0. \quad (4.1)$$

Here  $V$  – relative speed of the radiation source and detector, and  $R_L$  is determined as a distance covered by the signal arriving to the observation point from the source emitting this signal at some instance of time  $t_{in}$  in the distant past. A problem of finding the distance  $R_L$  may be solved if we find the signal propagation time, i.e. a time interval between the initial  $t_{in}$  and final  $t^{fin}$  points of the light cone generatrix in the observer's reference system. In this case  $R_L \stackrel{def}{=} ct$ , where  $c$  – speed of light,  $t = t^{fin} - t^{in}$ . In other words,  $R_L$  represents what is known as location (radar) distance.

To compare formula (4.1) with experimental data, one needs theoretical relations to express each of the quantities  $V$  and  $R_L$  as an explicit function of one and the same set of experimentally recorded radiation parameters. It is well known that a frequency shift is the most accurately measured quantity of this kind.

We know that, within the scope of SR, a relationship between the frequency shift

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \quad (4.2)$$

and relative speed (Doppler relativistic effect) is determined as a direct kinematic consequence of Lorentz transformations.

Let us demonstrate that the use of a group of special conformal transformations (SCT) representing, along with a Lorentz group (LG), a subgroup of the  $SO(4,2)$  conformal group for transformations of the 4-D pseudo-Euclidean space-time leads to the functional dependence of the electromagnetic signal propagation time on the frequency shift, in turn making it possible to express the distance  $R_L$  as an explicit function of the red shift  $z$ .

The metric quadratic form  $S^2 = x^\mu x_\mu (\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\})$  of Minkowski space is invariant with respect to Lorentz transformations (LT)

$$x'^{\mu} = L_{\alpha}^{\mu} x^{\alpha}, L_{\alpha}^{\mu} L_{\nu}^{\alpha} = \delta_{\nu}^{\mu},$$

meeting the condition

$$x'^{\mu} x'_{\mu} = x^{\mu} x_{\mu}. \quad (4.3)$$

Nevertheless, the Lorentz invariant  $S^2 = x^{\mu} x_{\mu}$  is not a scaling invariant. With respect to the transformations of SCT group

$$x'^{\mu} = \sigma^{-1}(a, x) \{x^{\mu} + a^{\mu} (x^{\alpha} x_{\alpha})\}, \quad (4.4)$$

where

$$\sigma(a, x) = 1 + 2(a^{\alpha} x_{\alpha}) + (a^{\alpha} a_{\alpha}) (x^{\beta} x_{\beta}), \quad (4.5)$$

$a^{\mu}$  – four-vector parameter, it behaves as follows:

$$x'^{\mu} x'_{\mu} = \sigma^{-1}(a, x) x^{\mu} x_{\mu}. \quad (4.6)$$

But as seen from (4.3) and (4.6), the requirements  $x'^{\mu} x'_{\mu} = 0$  and  $x^{\mu} x_{\mu} = 0$  in both cases are intercorrelated. This is a well-known fact of the light-cone equation invariance with respect to LT as well as SCT. The light-cone generating lines in both cases are, however, subjected to transformations, that should lead to variation of the observable frequency characteristics of electromagnetic signals on the transition between any reference system pairs associated with the corresponding transformation of coordinates.

First we consider a familiar situation with the relativistic Doppler effect.

For simplicity, the four-vector  $x^{\mu}$  is chosen in the form

$$x^{\mu} = \{x^0 = ct, x, 0, 0\} \quad (4.7)$$

and we limit ourselves to consideration of a single-parameter group of Lorentz boosts. It is clear that we refer to the longitudinal Doppler effect. For this case Lorentz transformations are of the form

$$x' = \frac{x - \beta ct}{\sqrt{1 - \beta^2}}, \quad t' = \frac{t - \beta \frac{x}{c}}{\sqrt{1 - \beta^2}}, \quad (4.8)$$

where  $\beta = \frac{V}{c}$ ,  $V$  – relative speed of motion for two inertial reference systems (IRS). In the case of an electromagnetic signal emission a light-cone

equation is of the  $x^2 - c^2t^2 = 0$  in nonprimed and  $x'^2 - c^2t'^2 = 0$  in primed RS. Substituting into (4.8) the expression  $x = ct$  (for definiteness, we select the case of the signal propagation in the positive direction of the light cone generatrix), with regard to  $x' = ct'$  we have

$$t' = t \left( \frac{1 - \beta}{1 + \beta} \right)^{\frac{1}{2}} \quad (4.9)$$

determining a linear deformation of the light-cone generatrices with Lorentz transformation. For small time increments  $\Delta t$  and  $\Delta t'$ , from (4.9) we can find that

$$\Delta t' = \Delta t \left( \frac{1 - \beta}{1 + \beta} \right)^{\frac{1}{2}}. \quad (4.10)$$

Identifying them with oscillation periods of the emitted ( $T_{emitted} = \Delta t'$ ) and observed ( $T_{observed} = \Delta t$ ) signals, respectively, and meaning by  $V$  a relative speed of the signal source and detector separation, from (4.10) we get

$$\frac{T_{observed}}{T_{emitted}} = \left( \frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}. \quad (4.11)$$

This well-known formula for the longitudinal Doppler effect is defining the frequency red shift. Considering the standard definition of (4.2), from (4.11) we derive the familiar relativistic expression connecting a relative speed of the radiation source and detector to a red shift value as follows:

$$V(z) = c \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}. \quad (4.12)$$

Now we direct our attention to SCT determined by formulae (4.4) and (4.5). As previously, we limit ourselves to treatment of the two-dimensional (1+1) version. The four-vector  $x_\mu$  is considered to be given by (4.7). According to [18], we select a four-vector parameter  $a^\mu$  of the form  $a^\mu = \{0, a, 0, 0\}$ , where  $a = -\frac{1}{2r_0}$ ,  $r_0$  – parameter with a dimension of distance. Transformations of (4.4) are conveniently written in the following noncovariant form:

$$x' = \frac{\xi x - \eta ct}{\xi^2 - \eta^2}, t' = \frac{t}{\xi^2 - \eta^2}, \quad (4.13)$$

where

$$\xi = 1 + \frac{x}{2r_0}, \eta = \frac{ct}{2r_0};$$

$$\sigma(a, x) \rightarrow \bar{\sigma}(\xi, \eta) = \xi^2 - \eta^2.$$

In the approximation  $\xi \approx 1$ ,  $\eta \ll 1$  from (4.12) we obtain

$$x' \cong x - \frac{c^2 t^2}{2r_0}, t' \cong t$$

that is associated with Galilean-Newtonian limit. Because of this, the parameter

$$\frac{c^2}{r_0} = w \quad (4.14)$$

actually denotes a uniform 3-D acceleration. From this it follows that SCT (4.4) in the approximation considered may be treated as a transformation from (unprimed) co-moving Lorenz reference system (RS) S to the (primed) noninertial RS  $S'$  moving at the uniform relative acceleration  $w$  in the  $+x$  direction. Also, note that, in the Galilean-Newtonian approximation, relations of (4.13) lead to the customary acceleration transformation rule on going from noninertial frame of reference to the inertial one

$$\frac{d^2 x'}{dt'^2} = \frac{d^2 x}{dt^2} - w. \quad (4.15)$$

It is readily seen that in RS  $S'$  the fact of noncoincident light signal propagation times in the forward and backward directions in the location-type experiments follows directly from the transformations of (4.13). This is demonstrated in Fig. 1.

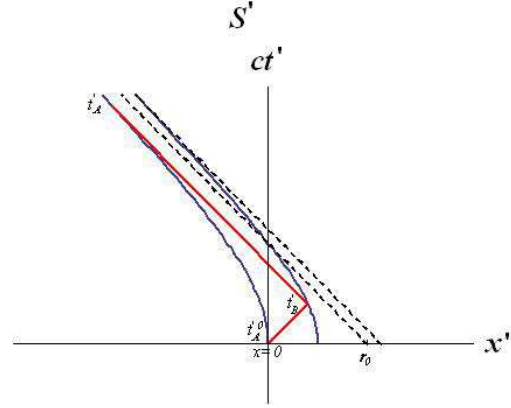
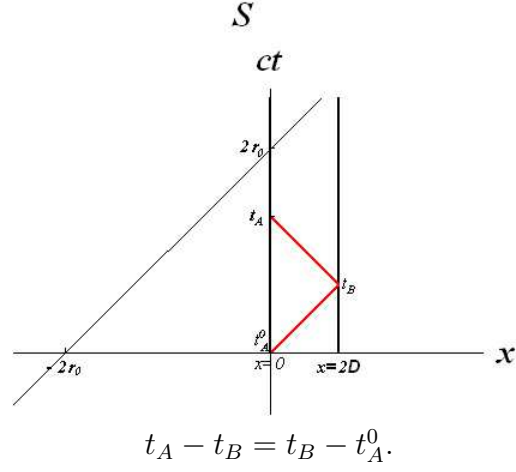


Fig 1.  $t'_A - t'_B = t'_B - t'^0_A$ .

Vertical straight lines (1) and (2) on the upper denote, respectively, observer's world line (1) and world line (2) of the spatially distant object that is stationary relative to the origin of RS  $S$ . As is known (see [18]), on a basis of transformation (4.13) these lines map into hyperbolae (1') and (2') in RS  $S'$  (on the lower). Dotted lines represent asymptotes of the corresponding hyperbolae.

World lines of the signal (light cone generatrices) are shown by thin lines. At transformations (4.13) these lines remain straight with a slope of

45° to the coordinate axes of both reference systems.

As seen, in RS S the (segment) lengths of generatrices are coincident:  $t_A - t_B$  and  $t_B = t'_A$ , whereas in RS  $S'$  the length  $t'_A - t'_B$  is always greater than  $t'_B - t'^0_A$ .

Next we consider transformations (4.13) for the case of a light signal propagation, i.e. we take account of the equations  $x_0^2 - x^2 = x'^2_0 - x'^2 = 0$ .

By substitution of  $x = \pm ct$  into (4.13) we can find that

$$x' = \pm ct \left(1 \pm \frac{ct}{r_0}\right)^{-1}, \quad t' = t \left(1 \pm \frac{ct}{r_0}\right)^{-1}, \quad (4.16)$$

from where it follows that SCT, retaining invariance of an equation for the light-cone, result in the following nonlinear transformation of its generatrices:

$$t'_\pm = t \left(1 \pm \frac{t}{t_{\text{lim}}}\right)^{-1}, \quad (4.17)$$

where  $t_{\text{lim}} = \frac{r_0}{c} = \frac{c}{w}$ , and signs  $(\pm)$  denote, respectively, the signal propagation in positive and negative directions with respect to the cone vertex. For differentially short periods  $\Delta t$  and  $\Delta t'$  from 4.17 we have

$$\Delta t'_\pm = \Delta t \left(1 \pm \frac{t}{t_{\text{lim}}}\right)^{-2}. \quad (4.18)$$

Next, identifying  $\Delta t$  with an oscillation period of the emitted signal ( $\Delta t - T_{\text{emitted}}$ ),  $\Delta t'$  – with the period measured at the observation point ( $\Delta t' - T_{\text{observed}}$ ) and assuming that the signal has been emitted in the distant past with respect to the time of observation (a lower sign in (4.18) should be chosen), we get the following expression to relate wavelengths of the emitted ( $\lambda_{\text{emitted}}$ ) and observed ( $\lambda_{\text{observed}}$ ) signals:

$$\lambda_{\text{observed}} = \lambda_{\text{emitted}} \left(1 - \frac{t}{t_{\text{lim}}}\right)^{-2}. \quad (4.19)$$

Owing to this, with the use of the standard red-shift definition  $\lambda_{\text{observed}}/\lambda_{\text{emitted}} = z + 1$ , we can find an expression for the signal propagation time from the instant of its emission to the time of observation in the form of the explicit function  $t(z)$

$$t(z) = t_{\text{lim}} \frac{(z + 1)^{\frac{1}{2}} - 1}{(z + 1)^{\frac{1}{2}}}. \quad (4.20)$$

Therefore, the location distance  $R_L$  may be determined from

$$R_L = R(z) = R_{\text{lim}} \frac{(z+1)^{\frac{1}{2}} - 1}{(z+1)^{\frac{1}{2}}}, \quad (4.21)$$

where  $R_{\text{lim}} = ct_{\text{lim}} = r_0 = c^2/w$ .

To relate  $R_{\text{lim}}$  to Hubble constant, we use expression (4.12) for  $V(z)$  and formula (4.21) for  $R(z)$ , then we write the ratio  $V(z)/R(z)$ :

$$\frac{V(z)}{R(z)} = cR_{\text{lim}}^{-1}f(z). \quad (4.22)$$

Here

$$f(z) = \frac{(z+1)^{\frac{1}{2}}}{(z+1)^2 + 1} \cdot \frac{(z+1)^2 - 1}{(z+1)^{\frac{1}{2}} - 1}. \quad (4.23)$$

As  $\lim_{z \rightarrow 0} f(z) = 2$ , it may be seen that in the limit  $z \ll 1$  from (4.22) a linear dependence takes place in the following form:

$$V = cR_{\text{lim}}^{-1}R$$

coincident with the standard form (4.1) of Hubble law. As a result, we get the relation connecting the parameter  $R_{\text{lim}} = r_0$  to Hubble constant  $H_0$  and to speed of light  $c$

$$R_{\text{lim}} = 2cH_0^{-1}. \quad (4.24)$$

Considering (4.14), the acceleration  $w$  is expressed in terms of  $H_0$  and  $c$  as follows:

$$w = \frac{1}{2}cH_0. \quad (4.25)$$

As seen, an explicit analytical expression representing in the approximation  $z \ll 1$  the relationship, characteristics for Hubble law, between the electromagnetic-signal emitter and detector separation speed, on the one hand, and the distance covered by this signal, on the other hand, may be derived within the scope of a conformal group of  $SO(4,2)$ .

The foregoing simplest derivation, making allowance only for the radial motion component is exclusively based on the use of explicit expressions

(4.9) and (4.17) for deformation of the light cone generatrices with respect to Lorentz transformations and special conformal transformations. Consequently, all the results presented in this Section are associated with pure kinematic manifestations of the conformal space-time geometry.

It is noticeable that in this case we succeeded in finding simple analytical expressions which incorporate the explicit red-shift dependence of the recession speed (4.12) as well as of the location distance (4.21). The function  $V(z)$  given by (4.12) is a well-known relativistic expression for the longitudinal Doppler effect. Thus, with the derivation of relation (4.12) given in this Section we could aspire to a more than methodological novelty.

As regards the expression (4.21) specifying the cosmological location distance in the form of an explicit function of the red shift, no other references to it in literature sources have been found by the author. Since a numerical value of the parameter  $R_{lim}$  in (4.21) has been determined in accordance with (4.24) in terms of the familiar physical constants, the expression for  $R_L = R(z)$  may be subjected to a direct experimental checking. It might be advisable that the proposed simple formula (4.21) attracted the attention of astrophysics specializing in the field of measurements of the distances to the cosmologically separated objects, even though the considerations involved look inadequately substantiated.

Note again that in the approach put forward a correlation with Hubble law is made in the approximation of small  $z$ . Just in this approximation the correlation (4.24) between the parameter  $R_{lim}$  and Hubble constant  $H_0$  has been found as well as relation (4.25) determining the uniform background acceleration  $w$  in terms of the constants  $H_0$  and  $c$ .

At the same time, it is seen that the approximation  $z \ll 1$  is in line with the requirement  $t/t_{lim} \ll 1$  in expression (4.20) and hence in the initial formula (4.17) determining the conformal time inhomogeneity. Let us write an expression following from (4.17) in the approximation linear with respect to  $\frac{t}{t_{lim}}$ :

$$t'_\pm = t \mp \frac{t^2}{t_{lim}}. \quad (4.26)$$

Here  $t_{lim} = c/w$ , and upper (lower) signs correspond to the signal propagation in the forward (backward) directions of the light cone generatrix. Taking account of definition (4.25) of the parameter  $w$ , from (4.26) we can find

$$t'_{\pm} = t \left( 1 \mp \frac{H_0}{2} t \right), \quad (4.27)$$

precisely repeating relations (3.10), (3.11) obtained in Section 3 within the scope of Einstein's clock synchronization procedure generalized to the case of expanding space. Naturally, after multiplication of both sides in (4.27) by  $c$ , we arrive at the formulae (3.12), (3.13) determining the distances covered by the signal in the forward and backward directions and derived in the preceding Section, within the clock synchronization procedure generalization to the case of the existing expansion.

It remains only to add, as a possible historic variant for the development of a relativistic concept, that, without doubt, the results of this Section could have been obtained by joint efforts of the authors of both SR versions – Einstein and Poincaré – should the discovery of the Universe expansion take place, e.g., in 1911. Indeed, the corresponding generalization of the clock synchronization at that time, in principle, might be performed by Einstein himself. Besides, it is well known that even in his paper written in 1905 Einstein [15] framed a relativistic theory of Doppler effect as a direct kinematic outcome of Lorentz transformations.

On the other hand, such an expert in the field of a group theory as Poincaré, who in his pioneer works “On a Theory of the Electron” [19] actually put forward a modern treatment of SR ahead of his time by several decades, in 1911 hardly could be ignorant of the conformal invariance of Maxwell field equations. Supposedly, for him a solution of the problem on the use of a group of the scaling-invariant special conformal transformations for the description of the light signal propagation process in the expanding space and hence on ascertaining the fact of the light-cone generatrices conformal deformation, with the following theoretical derivation of an expression for Hubble law and with the prediction of anomalous violet electromagnetic-signal frequency drift in the location-type experiments, would be a mere matter of technique.

## 5 Effect of accelerated Universe expansion without dark energy.

Let us consider a behavior of the function  $V(z)/R(z)$  over the whole interval of its application, i.e. in the semi-infinite interval  $0 \leq z < \infty$ . Using

(4.12) and (4.21) for the functions  $V(z)$  and  $R(z)$ , we can write the ratio  $V/R$  in the following dimensionless form:

$$\Phi(z) = \frac{V(z)}{H_0 R(z)} = \frac{1}{2} \frac{(z+1)^{\frac{1}{2}}}{(z+1)^{\frac{1}{2}} + 1} \cdot \frac{(z+1)^2 - 1}{(z+1)^{\frac{1}{2}} - 1}. \quad (5.1)$$

The function  $\Phi(z)$  over the interval  $0 \leq z \leq 5$  is graphically shown in Fig. 2.

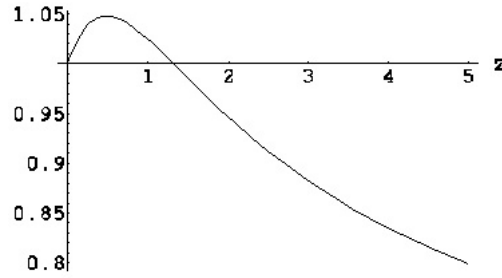


Fig 2.

As easily seen, for  $z \rightarrow \infty$  the function  $\Phi(z)$  asymptotically tends to the value that is equal to  $\frac{1}{2}$ . A horizontal line parallel to the  $z$ -axis is associated with the strictly proportional function  $V(R)$ . The function  $\Phi(z)$  is characterized by clear deviations from the line  $\Phi(z)=1$ , which are no greater than 5% over the interval  $0 \leq z \leq 2$ . It intersects the line  $\Phi(z)=1$  twice (at the points  $z=0$  and  $z \cong 1.315$ ) having a maximum at  $z_{\max} \cong 0.475$ .

To have a better understanding of the reasons for such a behavior of the function  $\Phi(z)$ , we represent it as  $\Phi(z) = \Phi_V(z)/\Phi_R(z)$ , where the functions

$$\Phi_V(z) = V(z)/c = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \cong \begin{cases} z(1-z) & \text{for } z \ll 1, \\ 1 - 2z^{-2} & \text{for } z \gg 1; \end{cases}$$

$$\Phi_R(z) = \frac{H_0 R(z)}{c} = 2 \frac{(z+1)^{\frac{1}{2}} - 1}{(z+1)^{\frac{1}{2}}} \cong \begin{cases} z \left(1 - \frac{3z}{2}\right) & \text{for } z \ll 1, \\ 2 \left(1 - z^{-\frac{1}{2}}\right) & \text{for } z \gg 1 \end{cases}$$

determine a speed of the signal source and detector separation and a distance covered by the signal expressed, respectively, in units of  $c$  and  $cH_0^{-1}$ .

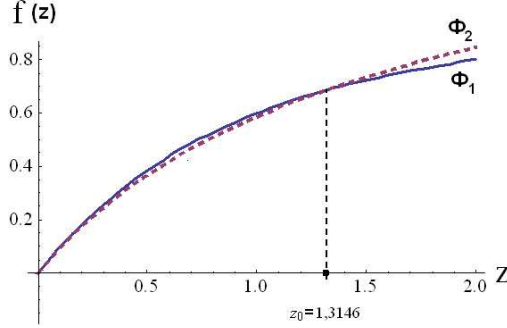


Fig 3.

The functions  $\Phi_V(z)$  (solid line) and  $\Phi_R(z)$  (dotted line) over the interval  $0 \leq z < 2$  are graphically shown in Fig. 3.

The values of both functions are coincident at the points  $z=0$  and  $z \cong 1.315$ . Within (beyond) this interval the inequality  $\Phi_V(z) > \Phi_R(z)$  ( $\Phi_V(z) < \Phi_R(z)$ ) is the case. As  $z$  is growing from zero to  $z_{\max} \cong 0.474$ , the speed increases more rapidly than the distance, the rate of increase of both functions is equal at the point  $z_{\max}$ , and then the distance is growing faster than the speed. For  $z \rightarrow \infty$  the speed tends to the limit that is equal to  $c$ , whereas the speed – to the limit  $R_{\lim} = 2cH_0^{-1}$ . It should be recalled again that both functional dependences  $\Phi_V(z)$  and  $\Phi_R(z)$ , and hence  $\Phi(z)$  as well, are of a pure kinematic origin.

Of course, the found behavioral features of the function  $\Phi(z)$  should be adequately interpreted from the viewpoint of physics. First of all, it is required to elucidate an extent to which the obtained theoretical dependence conforms to the data of astronomical observations. In the case under study, we consider the results of systematic recordings of the signals from the objects characterized by a fairly large red shift (close to  $z \approx 0.5$ ).

It is a common knowledge that just here in 1998-1999, owing to the advances in precise measurements of the intergalactic distances, a sensational discovery was made. Two independent research groups have discovered that the distances to the corresponding cosmologically distant sources are in fact greater than is required by the linear Hubble law. Naturally, this fact has been interpreted as an evidence for an increasing recession speed of galaxies, i.e. as an indication of the accelerated Universe expansion. As demonstrated by the experimental data, the transition from decelerated

expansion to the accelerated one has occurred in relatively recent times. The red shift value (or the “transition point”) has been determined from the observations as  $z_{\text{exp}} = 0.46 \pm 0.13$  (see [20,21]).

It is apparent that the expression (5.1) derived by us for the expansion law not only results in a linear dependence of  $V(R)$  in the limit  $z < 1$  but, in a surprising way, repeats the position of the experimentally detected extremum (point  $z_{\text{max}} \cong 0.475$ ). By our opinion, this bears witness to the fact that formula (5.1) describes the actually observed Metagalaxy expansion features at the modern stage (at least, beginning from small values of the red shift and up to  $z \rightarrow 2$ ). Clearly, the experimental data fitting procedures used in this work are distinct from those adopted in modern astrophysics for the construction of Hubble diagrams. At the same time, as has been noted previously, a simple analytical dependence on the red shift in our expressions containing only two fundamental numerical parameters ( $c$  and  $H_0$ ) is, in principle, approachable by direct experimental checking. But, provided the obtained values correctly simulate the observations, we still have to find an adequate physical interpretation for deviations of the function  $V(R)$  from linearity.

The conventional treatment is based on a cosmological model by Friedmann–Robertson–Walker (FRW). Within its scope, the dynamic measurand determining the Universe expansion is a deceleration parameter that gives a change in the expansion rate. This parameter connects the quantity and sign of the corresponding acceleration to the physical characteristics (density and pressure) of the gravitating matter. As this takes place, all the familiar types of the normal matter, hypothetic nonnuclear cold dark material (CDM) as well, contribute only to deceleration of the Universe expansion. Because of this, the experimentally detected acceleration of this process has lead to the knowledge that in the modern Universe there is a special substance: peculiar vacuum-like material state characterized by the presence of an internal tension (negative pressure). This hypothetical substance received the name *dark energy* or *quintessence*.

According to the estimates, a share of dark energy in the modern Metagalaxy may be as great as two thirds – three quarters of the total contribution made by all types of the gravitating matter. At the same time, we have to endow quintessence with the properties not only paradoxical from the viewpoint of the common knowledge but practically excluding the possibility of its direct experimental recording as a peculiar ontological substance. It is clear that such a situation cannot but stimulates a

search for alternative interpretations of the detected effect (see [22] and references herein).

Now we consider a possible interpretation for the expression (4.22) derived by us, remaining within the scope of the initial premises and without any special additional suppositions.

First we should recall that in the proposed pure kinematic approach no real sources of gravitation generating acceleration of bodies are represented at all. An essentially new element extending the SR kinematics is a nonlinear character of the conformal space-time transformations. This enables one to determine an explicit dependence between the distance and the red shift, to find an expression for Hubble law, and to establish a key relation of the conformal transformation parameter  $t_{lim}$  to Hubble constant  $H_0$  ( $t_{lim} = 2H_0^{-1}$ ). By this approach the “acceleration effect” arises in two limiting cases: on the light cone and far off the cone (nonrelativistic local approximation).

In the first case we have nothing else but acceleration mimicry of a kind. The original cause for the observed effect (anomalous frequency shift) is a conformal time inhomogeneity characterized in the approximation  $t/t_{lim} \ll 1$  by a uniform time acceleration [1,2] that is equal to  $\frac{d^2 t'}{dt^2} = H_0$  (corresponding mimicked acceleration  $W = c \frac{d^2 t'}{dt^2} = cH_0$ ).

In the second case  $w = \frac{c}{2}H_0$  characterizes an accelerated motion of the observer’s local co-moving (noninertial!) reference system. In noninertial RS itself this acceleration is opposite in sign, being background in character (i.e. it should be recorded in every point).

This conclusion is also supported by elementary dynamic considerations, within the scope of a single-particle Lagrangian model for motion in noninertial RS. As demonstrated in [23], determining the conformal-invariant action for a free large-mass particle in the standard way, one can have the following expression for the single-particle Lagrangian

$$L\left(\underline{r}, \underline{v}, t\right) = -mc^2 \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \left\{ \left(1 - \frac{r}{2r_0}\right)^2 - \left(\frac{ct}{2r_0}\right)^2 \right\}^{-1}, \quad (5.2)$$

from where, neglecting the terms  $\frac{v}{c}$ ,  $\frac{r}{2r_0}$ , and  $\frac{ct}{2r_0}$  with respect to unity, one can obtain

$$L_0\left(\underline{r}, \underline{v}\right) = \frac{m \underline{v}^2}{2} - \frac{mc^2}{r_0} r. \quad (5.3)$$

This expression complies with the Lagrangian describing a nonrelativistic motion of a large-mass particle under the action of the inertial force  $f$  given in the form

$$\mathbf{f} = -grad \left( \frac{mc^2}{r_0} r \right) = -\frac{mc^2}{r_0} \frac{\mathbf{r}}{r} = -\frac{mcH_0}{2} \frac{\mathbf{r}}{r}. \quad (5.4)$$

As the reference point is not assigned, the distance  $r$  may be set from an arbitrary chosen point in RS under consideration, the situation is completely in line with an idea of the presence of a negative background acceleration determined by the spatial expansion rate in accordance with  $w = \frac{1}{2}cH_0$ .

It is seen that, within the approach proposed, the experimentally recorded manifestations of the cosmological expansion are caused by a noninertial character of the observer's reference system. Actually, in this case there is no gravity. But the effect associated with the presence of the universal background acceleration may be interpreted proceeding from the equivalence principle in terms of the existing stationary background gravitational field. The more so that a negative sign of the acceleration is indicative of the decreased expansion rate in accordance with the standard dynamic pattern of the effect exerted by gravity on the behavior of the normal matter after the Big Bang.

Of particular importance is the fact that the mimic acceleration  $W$  computed from the observation data (i.e. based on experiments with propagation of electromagnetic signals) is exactly two times as great as the "mechanic" acceleration  $w$  directly determined from the conformal transformations in Galilean-Newtonian approximation (i.e. in the geometrically "flat" limit).

In a curious way, this situation is similar to the case of the light beam deflection by the gravitating center. As known, neglect of the space-time curvature by Einstein in 1911 (see [24]) has lead to the result half as great as that obtained by him four years later within the scope of GR (see [25]), to be in accord with the experimental data. Also, it should be noted that, conceptually and numerically, values of the acceleration  $w$  are extraordinary close to the value of the Milgrom *minimal acceleration* appearing as a fundamental parameter in the MOND (Modified Newton Dynamics, [26,27]) model that has been proposed as an alternative for the cold dark matter concept.

And, finally, note that the "equivalent" stationary background gravitational field with the intensity  $\sim cH_0$  present at every point of the Meta-

galaxy, in principle, may play a role of the alternative dark energy to do away with the present-day deficiency of the gravitating matter density in the Universe. Indeed, if we make use of a “naive” static field pattern and assume that the energy density of a background gravitational field may be found as  $\rho_F = \alpha \left( \frac{cH_0}{G} \right)^2$ , where  $\alpha$  – numerical parameter, a perfectly acceptable result,  $\rho_F = \frac{2}{3}\rho_c$ , where  $\rho_c = \frac{3}{8\pi} \frac{c^2 H_0^2}{G}$  – critical density, is achieved at  $\alpha = \frac{1}{4\pi}$ . It is understood that such considerations and estimates are not rigorous. A relativistically consistent description of noninertial reference systems necessitates the use of the GR techniques.

## 6 Concluding remarks.

The principal outcome of this paper is the demonstration that the observed features of the modern Universe expansion stage may be described consistently within the scope of the space-time conformal geometry, without the direct use of a pseudo-Riemann geometrical model of GR. Non-Euclidean character of the space-time manifold is presented in the proposed approach by conformal time inhomogeneity. Characteristically, the experimentally recorded local expansion effects in this case have to be interpreted as a manifestation of the noninertiality of observer’s reference system (“expanding” RS), in every point of which there is a background acceleration directed to the observation point, its numerical value being determined by the expansion rate (or by the observed value of Hubble parameter).

It is clear that such a theoretical model looks practically like a phenomenological one. But with this model there is a possibility not only to simulate the observed effects but also to have the experimentally checkable predictions. Moreover, even the fact that within this model (i. e. without resort to GR) an analytical dependence of the location distance on the red shift has been established and an explicit expression for the cosmological expansion law has been derived is undoubtedly of special interest.

Also, it is inferred that the necessary theoretical basis for description of the processes associated with nonstationarity of the spatial scales has been actually developed before the creation of GR – by 1911. In 1911, in principle, it could have been used to predict (later experimentally revealed) approximate linear dependence between the recession rate and the corresponding distance (Hubble law), intermediate maximum at the point  $z \approx 0.5$  (accelerated Universe expansion), and anomalous frequency shift

appearing in the location-type experiments (Pioneer anomaly).

It goes without saying that, due to a pure kinematic character of the approach, gravitational sources as such are lacking in the proposed expansion pattern. Nevertheless, they may be inferred on the basis of the Einstein equivalence principle as a background gravitational field, whose “intensity” as well as the associated acceleration is determined by the Hubble constant  $H_0$ .

In the proposed model the Hubble constant  $H_0$  acts as a parameter that, in combination with the speed of light  $c$ , determines the space-time dimensions for the observable region of the Universe. In this case the constants  $c$  and  $H_0$  are significantly differing in their status. The presence of the upper limit for speed is of a fundamental character, being organically involved in the SR bases. However, up to the present no arguments in favor of the existing constraints on the numerical value of Hubble parameter as inferred from a certain general physical principle have been put forward. By author’s opinion, the Maximum Tension Principle proposed by Gibbons in 2002 [28] looks very promising in this respect. By this principle, in nature there is an upper limit for the rate of a change in the energy-momentum (*maximum force*  $\left(\frac{dp}{dt}\right)_{\text{lim}}$  and *maximum power*  $\left(\frac{dE}{dt}\right)_{\text{lim}}$ ), and the corresponding numerical values of these limiting parameters are related to Einstein’s gravitational constant  $G/c^4$  as follows:  $\left(\frac{dp}{dt}\right)_{\text{lim}} = \frac{c^4}{4G}$ ,  $\left(\frac{dE}{dt}\right)_{\text{lim}} = \frac{c^5}{4G}$ .

Of course, in the absence of a developed theoretical concept based on the principle, only heuristic considerations and approximate quantitative assessments are possible. But such assessments lead to the results of particular interest. To illustrate, let us consider an expression for the acceleration determined as a ratio of maximal force  $F_{\text{max}} = \frac{c^4}{4G}$  to the Universe mass, the latter being determined as  $M_u = \rho_u V_u$ , where  $\rho_u$  – average density of the Universe mass,  $V_u$  – its volume. Assuming that  $\rho_u$  is coincident with the critical density  $\rho_c = \frac{3}{8\pi} \frac{H_0^2}{G}$ , whereas the volume is determined from  $V_u = \frac{4\pi}{3} R_u^3$ , where  $R_u = cH_0^{-1}$ , we can derive for  $M_u$  the following expression:  $M_u = \frac{1}{2} \frac{c^3}{GH_0}$ . As a result, for the sought quantity we have  $w = \frac{F_{\text{max}}}{M_u} = \frac{1}{2} cH_0$ , that is surprisingly coincident with the background acceleration quantity found by us. Thus,  $H_0$  observed presently looks like a limiting value (or close to the limit), the limit, due to a fundamental character of the constant  $c^4/G$ , being determined exclusively by the matter amount in the Metagalaxy.

Even though the coincidences of this kind should not be overestimated, it is not improbable that just a consistent allowance for the constraints imposed by the Maximum Tension Principle sets forth the way to possible modification of the traditional GR for description of the gravity in extreme conditions.

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